Fracture, Longevity, and Damage Tolerance of Graphite/Epoxy Filamentary Composite Material

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Fracture, longevity, and damage tolerance are among the barriers to be surmounted before full exploitation of the graphite/expoxy filamentary composite materials can occur. Fracture defines the conditions which will precipitate catastrophic failure, damage tolerance is the ability of the airframe to resist failure in the presence of damage, and longevity is the life of the airframe. The present state of knowledge with respect to these three characteristics is described. Static damage modes are not the same as those under cyclic loads. Fatigue is not a meaningful description of the behavior of the brittle filaments. Longevity is a natural outcome of designing for damage tolerance. Evidence is presented which suggests that damage tolerance of graphite/epoxy is as good if not better than the metals. Research emphasis should be directed toward the propagation of damage under cyclic loads and the amount of damage which will cause catastrophic failure.

Introduction

IT has been found that graphite/epoxy possesses a sufficient number of desirable attributes to be considered a "balanced-material." These attributes include: 1) excellent strength characteristics; 2) material costs which are competitive with the metals; and 3) despite being labor intensive fabrication is achievable at a tolerable cost and service experience has generally been good. These obvious structural attributes can be explorited more fully, however. A measure of full exploitation is that the advanced filamentary composite materials comprise at least 50% of the structural weight of aircraft, such as transport category types or high performance military fighters.

Definitions

Fracture, longevity, and damage tolerance are among the issues which must be addressed by the engineer if adequate structural integrity is to be achieved. Fracture is defined herein as the catastrophic failure which ensues when the stress is sufficient to cause the rapid growth of "flaws." In metals the flaws are primarily "cracks" and the appropriate terminology is to say that metals are "notch sensitive." Filamentary composites are "notch sensitive" to slits and holes, i.e., holes affect the static strength of filamentary composites as do cracks in metals. The word "longevity" is purposefully used instead of "fatigue." For safe life structures, the initiation of cracks was considered to be the end of life and hence "fatigue" had a useful connotation. But the structural integrity of a metal structure with cracks now can be assessed and a useful life with adequate flight safety can be prescribed. A proposal is herein offered for the abolition of the terms "fatigue" and "fatigue life" from the lexicon of the aerospace engineer. In it's place the author offers the term "longevity," which has a much more positive connotation. The engineer is designing the aerospace vehicle for a structural service life during which all of the integrity issues are satisfied, i.e., he/she is designing "longevity" into the structure.

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Another reason for abolishing the term "fatigue" is the evolving structural design philosophy of damage tolerance. Damage tolerance is the ability of the airframe to resist failure even in the presence of undetected flaws, cracks, or other damage for a specified period of unrepaired usage.[1] This structural design philosophy, first embodied in the military specification, Mil-A-83444, "Airplane Damage Tolerance Requirements," leads to designs with the greatest level of structural integrity. It is of interest to note that the following phrase is contained in Mil-A-83444: "...not intended to be directly applicable to advanced composite structures..." The reason for this phrase is that the dominant mode of damage in a metallic structure is a crack, and a crack not only has a well-defined mathematical representation but is also well characterized experimentally. A crack is a crack is a crack, whether the material is aluminum or titanium or steel, and the same methodology can be applied. Such a state of the art does not yet exist for filamentary composite materials. Thus, the inclusion of the above disclaimer in Mil-A-83444 is a recognition of the level of understanding which existed at the time this specification was revised (circa 1974). The Federal Aviation regulations were also revised in acknowledgement of the increased levels of integrity which could accrue from the philosophy of damage tolerance. This occurred in 1978 with a revision of rule 25.571 and the issuance of Advisory Circular 25.571-1, entitled "Damage-Tolerance and Fatigue Evaluation of Structure." With damage tolerance concepts and with an inspection program based upon damage tolerance, the level of structural integrity of airframe need not fall below specification levels. There may be a point where the structural repairs and the frequency of inspections are economically unacceptable but, insofar as flight safety is concerned, the airframe need not have a finite life. Thus, the practical longevity of an airframe is dictated by inspection and repair costs or by technical obsolescence, not by flight safety.

The quantification of damage tolerance requires a knowledge of the fracture characteristics of the G/E composites and this is accomplished by specifying the state of stress which will cause catastrophic failure to initiate from "flaws." Fracture defines the bounds of damage tolerance by determining the conditions under which the structure becomes intolerant. In order to prescribe the interval between inspections, the longevity of the structure must be assessed, i.e., the manner and rate of growth of damage to critical sizes under cyclic loads must be determined.

Fracture

For unflawed specimens the tensile strengths are exceptional and, when normalized by the density, the specific strengths are much superior to the high strength alloys (e.g., aluminum 7075-T6, titanium 6A1-4V, and 300M steel alloys used in aerospace vehicles). Similarly, the specific moduli of elasticity of practical G/E laminates are much superior to the metals. We can dramatize this data by using units of psi (pounds per square inch) for strength and modulus and pci (pounds per cubic inch) for density. This yields specific strengths and modulus in units of length, making it germane to say that the G/E filamentary composite materials are "miles better" than the metal competition. Table 1 compares uni-directional G/E properties with the metal alloys, while Table 2 lists the specifics for a number of angle-plied laminates. It should be noted that the quasi-isotropic lay-up $[0/\pm 60]s$, which represents an inefficient usage of filamentary materials, is still "miles better" than the metal competition.

The fracture behavior of the G/E filamentary composites presents many varied and exciting challenges. Figure 1 is a post-mortem photograph of the tensile fracture appearance of a $[+-30]s$ angle-plied laminate.[2] The fracture faces are "clean", i.e., definable by a single straight line over most of the cross section. Figure 2 is a post-mortem photograph of the tensile fracture appearance of a $[0/+-30]s$ laminate. In contrast to Fig. 1, the fracture faces are not easily defined in a geometric sense because the broken ends of the filaments do not lie on the same plane. The mode of failure also exhibits some delamination, which is another contribution to the unkempt appearance of the fracture. Predictions of the fracture stresses or of the fracture appearances are not within the current state-of-the-art.

An example of the varied behavior of laminates is shown in Fig. 3 where the tensile fracture of the family $[00/+\theta]s$ containing a 6.35 mm diam hole is shown.[6] This family divides itself into two groups, one wherein the angles plies are at angles less than 30 deg and the other at angles greater than 30 deg. Three of the five specimens with angle plies at 30 deg behaved in one mode and the other two behaved in the other mode. In one mode, as shown in Fig. 3, the stress-strain diagrams are essentially linear to failure, whereas in the other mode the stress-strain diagrams contain a kink near the ultimate stress points. Some mechanism, as yet unexplained, apparently causes some of the $[00/+-30]s$ specimens to fail at an appreciably lower stress, whereas in the other group some other mechanism enables the specimens to surpass this lower stress with a readjustment of its stress-strain behavior.

In ductile metals the plasticity of the stress-strain diagram readjusts the nonuniform stress distribution caused by a hole in a tension member such that the ultimate strength can be calculated with reasonable accuracy by using the unflawed strength applied to the net section. This is not the case for filamentary composite materials. Stress concentrations for an

Table 1 The competitors

	Specific strength (miles)	Specific stiffness (miles)
2024 – T3 A1	11	1600
7075 – T6A1	13	1600
7175 – T73A1	11	1600
Ti6A1 – 4V	13	1600
300M steel	16	1600
G/E uni	65	5100
K/E uni	84	3500

Table 2 Angle-plied properties

	Specific strength (miles)	Specific stiffness (miles)
7175 – T73A1	11	1600
G/E [0]	65	5100
G/E $[0/\pm 30]_s$	37	3200
G/E $[0/\pm 45]_s$	31	2300
G/E $[0/\pm 60]_s$	32	2100
G/E $[0/\pm 90]_s$	29	2100

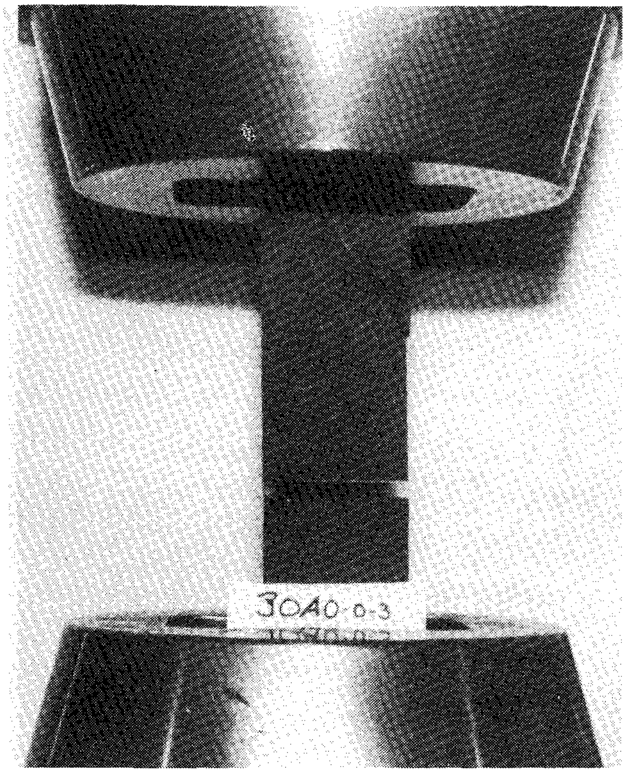

Fig. 1 Post-mortem of $[\pm 30]_s$ laminate.

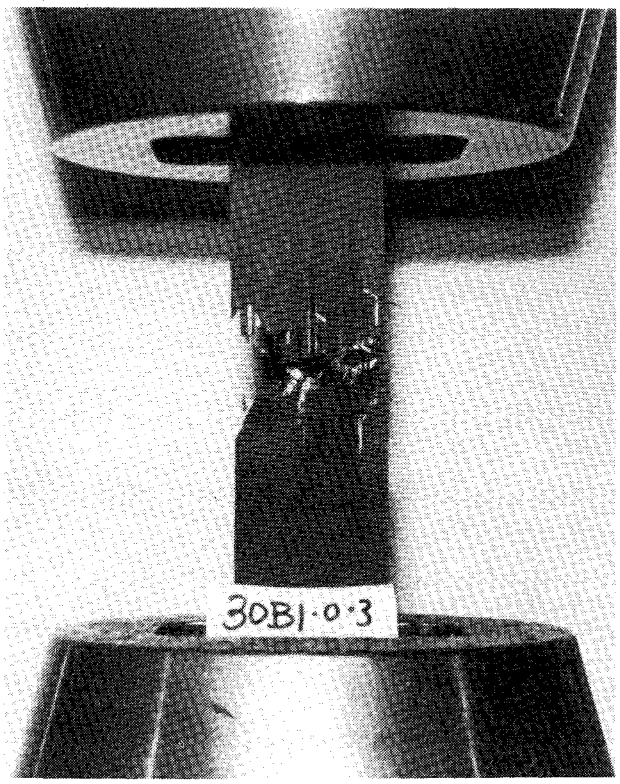

Fig. 2 Post-mortem of $[0/\pm 30]_s$ laminate.

open hole vary from about 7 to slightly less than 3 based upon orthotropic elasticity theory.[7] The effect of holes on the tensile strength is shown in Fig. 4. Three sets of data are shown, one set by Whitney-Nuismer[8], one by Walter et al.[9] and one by Waddoups et. al.[10] Each set of data has been normalized by its respective virgin strength. If the composite laminates behaved in the same manner as ductile metals, then the experimental data would plot on the line labeled "line of net stress." Fortunately, even through their stress-strain diagrams are essentially linear to failure, these filamentary composites do not respond to stress concentration as would a true brittle material. The experimental data indicates that the G/E filamentary composite materials are "notch sensitive" to holes. It is interesting to note that a similar notch sensitivity effect on compression strength has been observed by Rhodes and Mikulas.[11]

The effect of holes on the tensile strength of various laminates has been studied extensively by Lagace.[2] Some of his data are shown as log-log plots of fracture strength vs. hole diam in Fig. 5. The method of presentation is motivated by linear elastic fracture mechanics wherein the influence of cracks on strength is correlated to "fracture toughness" and the square root of the crack length. On a log-log plot, the fracture strength of a metal with cracks plots as a straight line with a slope of 0.5. The goodness of fits of the data shown in Fig. 5 are better than .95. The slopes of the lines range from .10-.34, with most of the laminates exhibiting a slope in the neighborhood of .28. Lagace observed that stacking sequence and hole diameter determine whether or not delamination is a major characteristic of the failure. For those laminates which do not exhibit significant delamination, the slope is approximately .28. In these laminates the predominant modes of failure are the rapid fracture of the graphite filaments. The $[0/+-75]s$ laminate showed delamination for all hole sizes while the $[0/+-90]s$ laminate showed a mixed behavior where the specimens with the two smallest hole sizes exhibited delamination while the two larger holes did not induce delamination. Delamination is evidence that the fracture is comprised of more than one mechanism and is the reason for the slope being different than .28. It is informative to correlate the data as plotted in Figs. 9 and 10 according to the relationship shown in Table 3 where f is the gross area fracture stress in (mega pascals) MPa, g is the virgin strength in MPa, 2a is the hole diameter in millimeters, m is the slope of the log-log plot, Hc is the composite toughness, and A is the ratio of Hc to g. The values are shown in Table 3.

Another behaviorial mode of filamentary composite materials is the effect of a hole on the fracture strength of a unidirectional laminate. Under a monotonically increasing load applied along the filament directions, the first signs of distress in a tensile coupon containing a hole are the appearance of "splits" emanating from the edge of the hole. These "splits" occur as a result of cracks in the matrix and are not disbonds between the filament and the matrix. As the load is increased, the splits grow in length so that at failure the specimen is divided into three regions with the central region containing the hole. This central region does not carry any of

Table 3 Toughness parameters of $[0/\pm\theta]_s$ and $[\pm\theta/0]_s$ laminates

	M	A	g MPa
$[0/\pm 15]_s$	.29	.929	1083
$[0/\pm 30]_s$	.31	.807	945
$[0/\pm 45]_s$	.29	.800	787
$[0/\pm 60]_s$	.34	.992	814
$[0/\pm 75]_s$	.22	.967	733
$[0/\pm 90]_s$	.10	.905	732

Note: $f = gA(2a)^{-m} = Hc(2a)^{-m}$, $A = \dfrac{Hc}{g}$

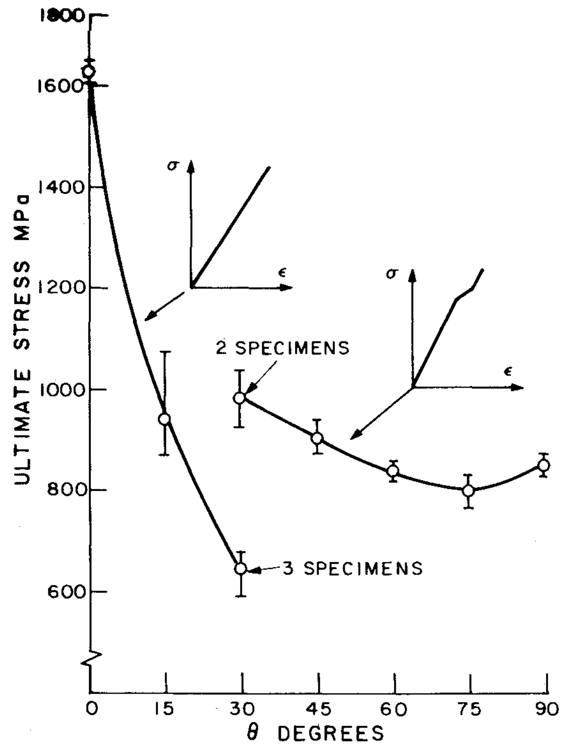

Fig. 3 Ultimate stress of $[0_2/\pm\theta]_s$ laminates with a hole.

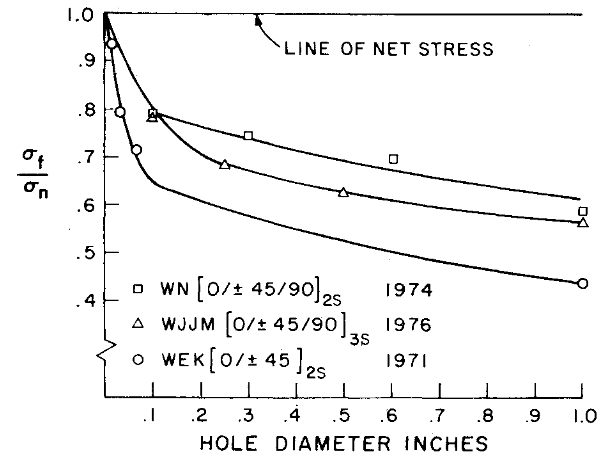

Fig. 4 Notch sensitivity to holes.

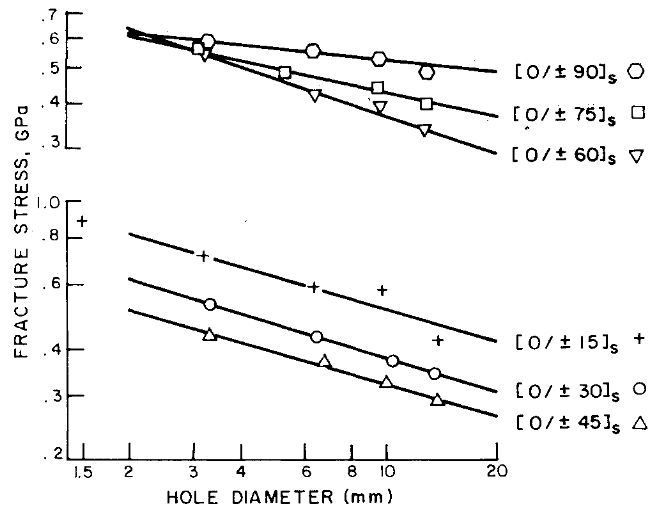

Fig. 5 Fracture with holes $[0/\pm\theta])_s$.

the ultimate load and the two regions on either side of the central region behave like two separate specimens. In other words, the ultimate load of a unidirectional specimen containing a hole can be predicted by using the virgin strength and the net cross-section area. Thus, in contrast to the laminates which contain angle-plies, the ultimate strength of a unidirectional laminate is not "notch-sensitive" to a hole or a slit. The stress at which the split develops is a function of hole diameter.[12] Strain measurements near the hole indicate that the stress concentration is eliminated by the onset of the split and that the central region of the specimen is not strained once the split becomes fully developed.[13]

There is evidence that under tension-tension cyclic loads the longevity of filamentary composites is essentially limitless. To use metal terminology, the S-N diagram under tension-tension cyclic loads is almost flat, i.e., the "fatigue" life is practically limitless.[14] The result should not be surprising because the graphite filaments, which provide the strength of the composite, are brittle. Hence, the strength of a filament depends upon the presence of the largest flaw. There is no evidence that cyclic loads create flaws or enlarge flaws in the filament. Rather, the emphasis on the longevity of filamentary composites has shifted from concern with tension-tension cyclic loading to compression-compression.

Stress concentrations are minimized in metal structures to prevent the early onset of cracking under cyclic leading. Thus, for metals cracks are the dominant form of damage which to be avoided. In filamentary composite materials, there are many competing failure modes. Furthermore, the failure modes, as determined by post mortems, are not in general the same under static loads as under compression-compression cyclic loads. Additionally, the presence of a "flaw" such as a hole or slit also alters the mode of failure. This is depicted by means of sketches in Fig. 6. There is a growing body of data which indicates that the dominant mode of damage in the presence of a flaw under compression-compression cyclic loading is delamination.

Damage Under Cyclic Loads

It is a fact that the areospace industry has used S-N diagrams and some variants of Miner's Rule to great advantage. Given a sufficiently large data base, which is tailored for a narrowly specified cyclic load spectrum, the use of Miner's Rule applied to S-N curves can result in a vehicle with adequate longevity. The key feature is that constant amplitude data is used to determine the so-called "fatigue life" under the actual service usage. In order to be assured of the desired life, a scatter factor of 4 is used, i.e., the design and test life is 4 times the desired life. Miner's Rule is not rooted in the physics of the fatigue phenomenon and has been shown to give wrong answers in many realistic situations. Thus, it seems inappropriate to embark on a program to determine S-N curves for the composite materials. What is required is the methodology to predict the growth of damage and this is available for metals in the discipline known as fracture mechanics. The modern approach to structural integrity, i.e., damage tolerance, does not even require the ability to predict the initiation of damage; it is assumed that damage exists and that design is such that the damage will not grow to the level where catastrophic failure with ensue. There is no methodology available for the prediction of damage growth in filamentary composites. At this point in time, the kinds of damage are insufficiently categorized to allow for precise mathematical descriptions.

One form of damage is a "split", which is a crack running parallel to the filaments. As shown earlier, a hole in a unidirectional composite induces splits which emanate from the edge of the hole. Tests[6] have been run at cyclic loads below the level at which a monotonic application will initiate a split. In Fig. 7 the growth of four splits in a unidirectional specimen containing a .125 in. diam hole is shown. The cyclic load varied between 8 and 80% of the stress which statically would initiate a split. The growth of the four branches of splits are shown in a semi-log plot and the two branches on either side have been added to show the single splits which eventually formed on either side. Of special interest is the relatively low number of cycles at which the split is initiated. As can be seen, the coalesced left and right splits eventually respond to cyclic loads at essentially the same rate. Another series of tests on the same phenomena have been conducted on eight ply unidirectional laminates.[12] The plots of splitting stress vs hole size reveal a slope sufficiently close to 0.5 to correlate this mechanism of damage with the methodology of linear elastic fracture mechanics of isotropic materials.

Under compression-compression cyclic loading the damage which occurs is delamination. There are experimental difficulties in conducting these kinds of experiments because the prevention of buckling is a major requirement. A solution has been to use sandwich specimens wherein the core stabilizes the laminate. There is presented in Fig. 8 a series of photographs which show the development of delamination in the vicinity of a 0.5 in. diam hole in a laminate which has been loaded as the

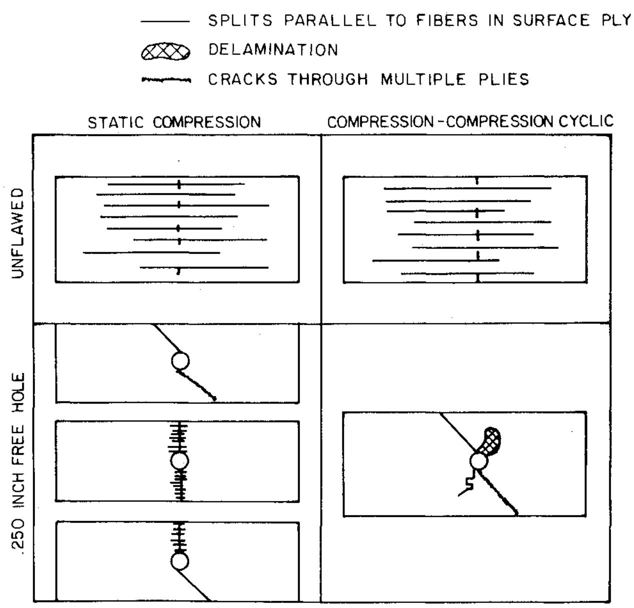

Fig. 6 Comparison of failure modes $[0/\pm 45]_s$.

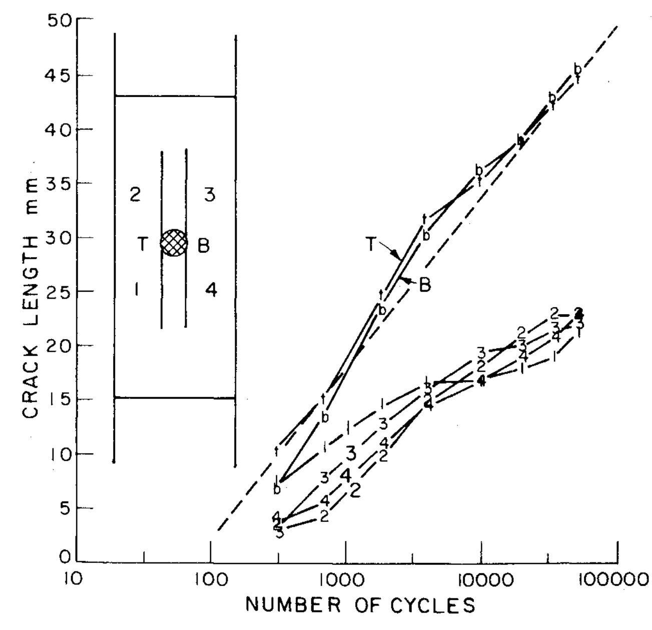

Fig. 7 Split length vs log cycles.

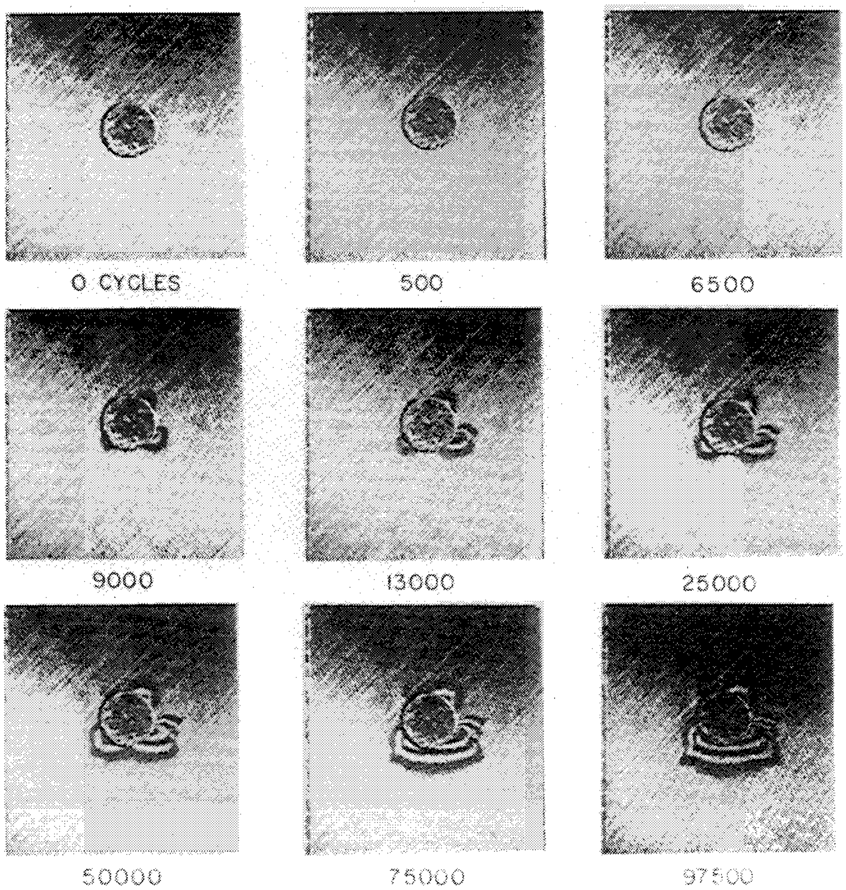

Fig. 8 Growth of delamination.

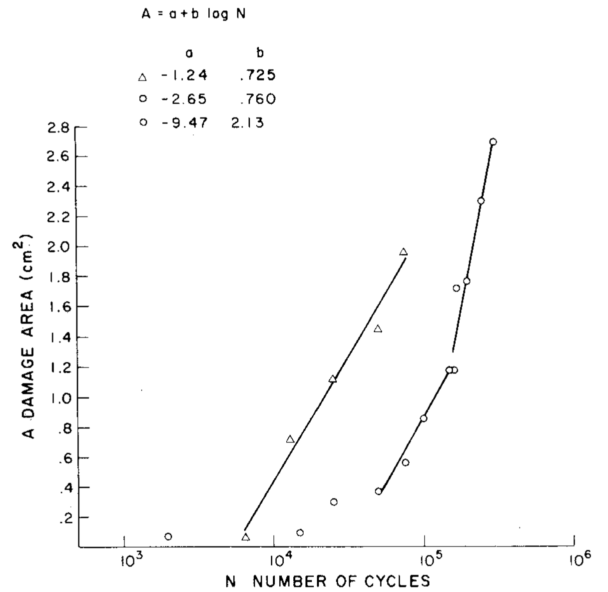

Fig. 9 Damage area vs log cycles.

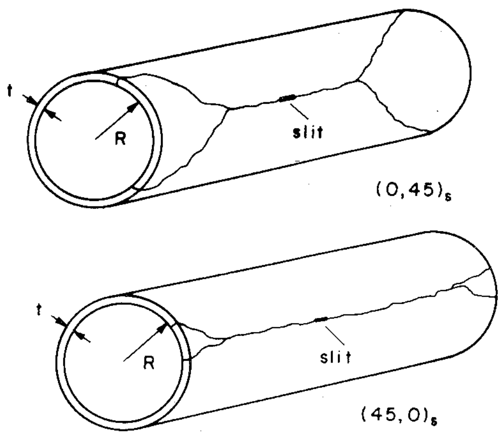

Fig. 10 Failure modes of unreinforced cylinders.

facing of a sandwich compression panel.[16] By means of a Moire interferometry out-of-the-plane technique, it has been detected that the delaminated region under the compression load buckles away from the initially planar laminate. In these photographs the maximum stress applied to the laminate was 386 MPa; the minimum was 10% of the maximum and the static compressive strength was 455 MPa. The longevity of this particular specimen was slightly over 98,000 cycles. By projecting the negatives onto a screen, the damage area was measured during the course of the cyclic loading. Some typical results are plotted in Fig. 9. It was noted that the initiation of damage occurred very early in the test, of the order of thousands of cycles, even though the final failure of the laminate occurred much later.

Damage Tolerance

Another aspect of the fracture characteristics of filamentary composite is the response of pressurized cyclinders to the presence of through-the-wall cracks. Twelve-in. diam cyclinders approximately 30 in. long were fabricated from four plies of a balanced woven cloth and were made pressure tight by means of aluminum endcaps.[17] While under pressure, a knife with a blade of a prescribed width was quickly inserted through the wall. If catastrophic failure did not occur, this slit was sealed, the cylinder rotated, and the next size of knife used. This process was repeated with blade sizes in increments of .25 in. until the cylinder failed. Hoop strains at failure ranged from about .0016-.0026 in. The failure modes are shown in Fig. 10 for the two configurations tested; one had the outside plies oriented with filaments parallel and perpendicular to the longitudinal axes of the cyclinders, with filaments of the two inner plies at 45 deg to the axis of cylinder. The other configuration had the two inner plies oriented with filaments parallel and perpendicular to the axis, with the outer plies oriented at + −45 deg. In both configurations, the failures were catastrophic. The configuration labeled (45,0), which had the filaments parallel and perpendicular for the two inner plies, shattered into smaller fragments than did the other. Even though the shell wall bending stiffnesses for the two configurations are quite different, the failure pressures are essentially the same.

The basic fracture characteristics of the four plies of cloth were determined by means of flat coupon specimens and the fracture stresses in the presence of slits correlated according to the formula shown in Fig. 11 (Ref. 18). The data to determine the constants in the formula were obtained from specimens in which the slits were .125, .25 and .50 in. in length. Once the slit is cut into the cylinder wall, the internal pressure causes local bending and there is no longer a simple membrane state of stress. The local bending increases the severity of processes which cause catastrophic failure. Two methods were used to account for this severity, both based upon linear elastic fracture mechanics. The first, used by Folias,[19] is based upon

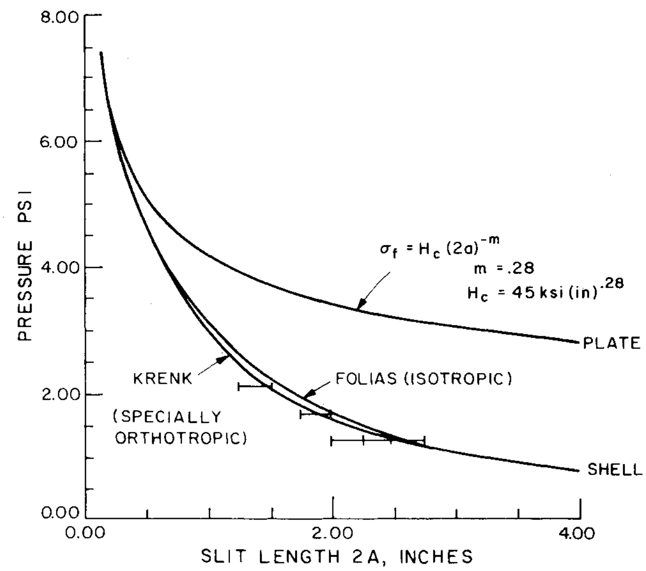

Fig. 11 Fracture of pressurized cylinders (4 plies G/E cloth).

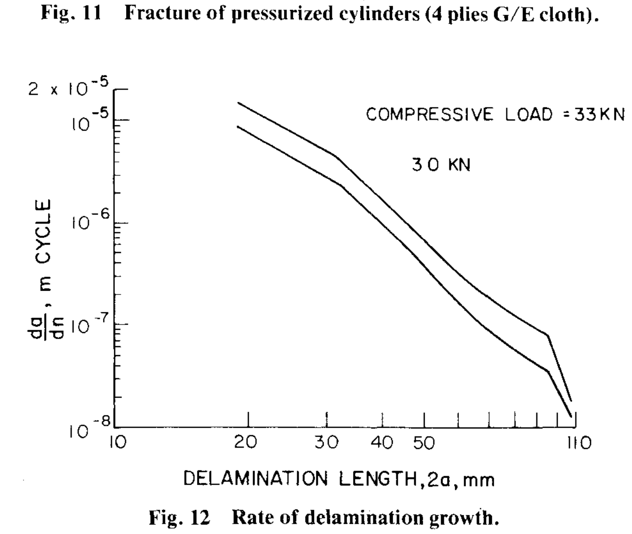

Fig. 12 Rate of delamination growth.

the material being isotropic, while the second, used by Krenk,[20] is based upon the material being specially orthotropic. As can be seen, the differences between the Folias and Krenk corrections are not significant. The good agreement obtained by correcting the extrapolating flat coupon data is seen in Fig. 11.

There is evidence that structures fabricated from the filamentary composites have the potential to be more damage tolerant than metal structures. 1) Under cyclic loads, a hole in a metal structural member eventually induces a crack (which in earlier times was considered the end of life, i.e., the fatigue life). Before the appearance of the crack, the elastic stress concentration is three but, once the crack appears, the stress concentration is theoretically infinite. As the crack grows, the rate of the growth of the crack increases. Thus, in metallic structures stress concentration increases from nominal values like three to infinity; stress intensity, which is a measure of the conditions at the tip of the crack, increases and the rate of crack growth increases exponentially.

2)The static strength of filamentary composite structures is severely degraded by the presence of a hole. There is a stress concentration at the edge of the hole which can vary from about 3-7, depending upon the lay-up orientation of the plies. Under cyclic loads damage is initiated at the hole and once this damage has been created, the laminate in the vicinity is no longer "whole". The initial stress concentration no longer exists and, to a degree, alternative load paths are intrinsically developed by the damaged laminate. As the amount of damage increases, there is evidence that the rate of growth decreases. The best correlation of the growth of the splits (See Fig. 7) is to plot the length of the split against the log of the number of cycles. Observe that in this figure the data correlates well as a straight line in the semi-log plot. Thus, with split length being proportional to log N, where N is the number of cycles, the rate of growth of the split length becomes inversely proportional to the number of cycles. This dependence is also seen in Fig. 9. This data seems to indicate there are regions of the life wherein the growth is reasonably approximately by a straight line on a semi-log plot. Thus, damage as measured by delaminated area also has a rate of growth which is inversely proportional to the number of load cycles.

3) An informative model for the study of the growth of delamination has been proposed by Whitcomb.[21] Experimental data from this model is shown in Fig. 12 as a plot of the rate of growth per cycle as a function of delamination length. It is observed that the rate of growth decreases rapidly with the length of the delamination. This concurs with what we would intuitively believe to be the case since the peeling action of the buckled region is a function of the local curvature and the curvature does decrease with increasing length of the delamination. Metal crack growth data plotted to the same scales used in Fig. 12 lies on a line with a positive slope, i.e., rate of crack growth increases with increasing crack length.

4) Calculations based upon the data shown in Fig. 11 show that a fuselage 80 in. in diameter fabricated from four plies of cloth will not burst at an internal pressure of 10 psi until a longitudinal slit of approximately 8 in. is cut through the wall. An aluminum fuselage fabricated from a .032 in. thick 2024-T3 aluminum material will burst with a crack of about 6 in. Thus, the .032 in. aluminum fuselage, which has the same weight as the G/E cloth one, is not as tolerant of damage. These calculations are based upon a fracture toughness of 100 ksi, root in. for 2024-T3. Even if the wall thickness of the aluminum is increased to .040 in. the critical crack size increases only to 7 in., which is still less than for the G/E fuselage.

As the community gains more experience with filamentary composite materials, design schemes to minimize the notch sensitivity will be invented. The present high level of structural integrity of metal airplanes was attained with a great deal of trauma and, unfortunately, with some catastrophic service failures.

Concluding Remarks

This author believes damage tolerance should be the primary structural design philosophy. Herewith are observations and recommendations to achieve the methodology required to apply damage tolerance to G/E filamentary composite airplanes.

1) Damage modes under static loads cannot be used as models for damage initiation or propagation under cyclic loads.

2) The filaments are brittle and, hence, fatigue is not a meaningful description of what happens under cyclic loads.

3) S-N types of data may be useful for comparing different material systems but are not useful for designing in accordance with a damage tolerance philosophy.

4) Longevity is a natural outcome of designing for damage tolerance.

5) There is evidence that damage in the form of filament breakage and microcracks in the matrix occurs at relatively low stress levels, even though the number of cyclic loads is small. This is further justification for striking "fatigue" from the lexicon because damage initiation is the historical definition of "fatigue life".

6) The research emphasis should be on the propagation of damage under cyclic loads and the amount of damage which will cause catastrophic failure.

7) Experiments and analyses should be aimed at understanding the damage initiation and accumulation in the epoxy material and at the fiber/matrix interface.

Acknowledgments

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